Einstein's steady-state cosmology

Last year, a team of Irish scientists discovered an unpublished manuscript by Einstein in which he attempted to construct a "steady-state" model of the universe. **Cormac O'Raifeartaigh** describes the excitement of finding this previously unknown work

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One afternoon in the summer of 2013, I was sitting in my office thinking about going for a swim. It was a beautiful day and I was becoming weary of the job in hand. That job consisted of analysing a paper called "On the cosmological problem of the general theory of relativity", which Albert Einstein had published in April 1931 (*Sitzungsberichte der König. Preuss. Akad. Wiss*. 235). I had found some numerical anomalies in Einstein's paper, and was hoping that a study of his original manuscript might shed some light on the matter. Many such documents can nowadays be viewed on the Einstein Archives Online maintained by the Hebrew University of Jerusalem, and sure enough, there it was in plain view (figure 1).

Or was it? The title and the opening paragraphs of Einstein's manuscript seemed familiar, but as I read on, it became more and more clear that, whatever this document was, it was not a draft of Einstein's *Sitzungsberichte* paper of 1931. For a start, Einstein's "cosmological constant" λ was evident in the equations of the manuscript (figure 2), while this term was almost completely absent in the published paper. (Indeed, the *Sitzungsberichte* paper is renowned as the first occasion on which Einstein formally banished the cosmological constant, in light of emerging evidence for the expansion of the universe.)

And then, the written text of the manuscript also contained some very puzzling statements. Chief among these was the assertion: "*Die Dichte ist also constant und bestimmt die Expansion*" ("The density is therefore constant and determines the expansion", see figure 2). At that point, all my thoughts of a swim were banished. Instead, I leapt from my chair and, printout in hand, ran down the corridor to the office of my colleague Brendan McCann, a mathematics lecturer with a formidable command of German, exclaiming "I think I've found something!"

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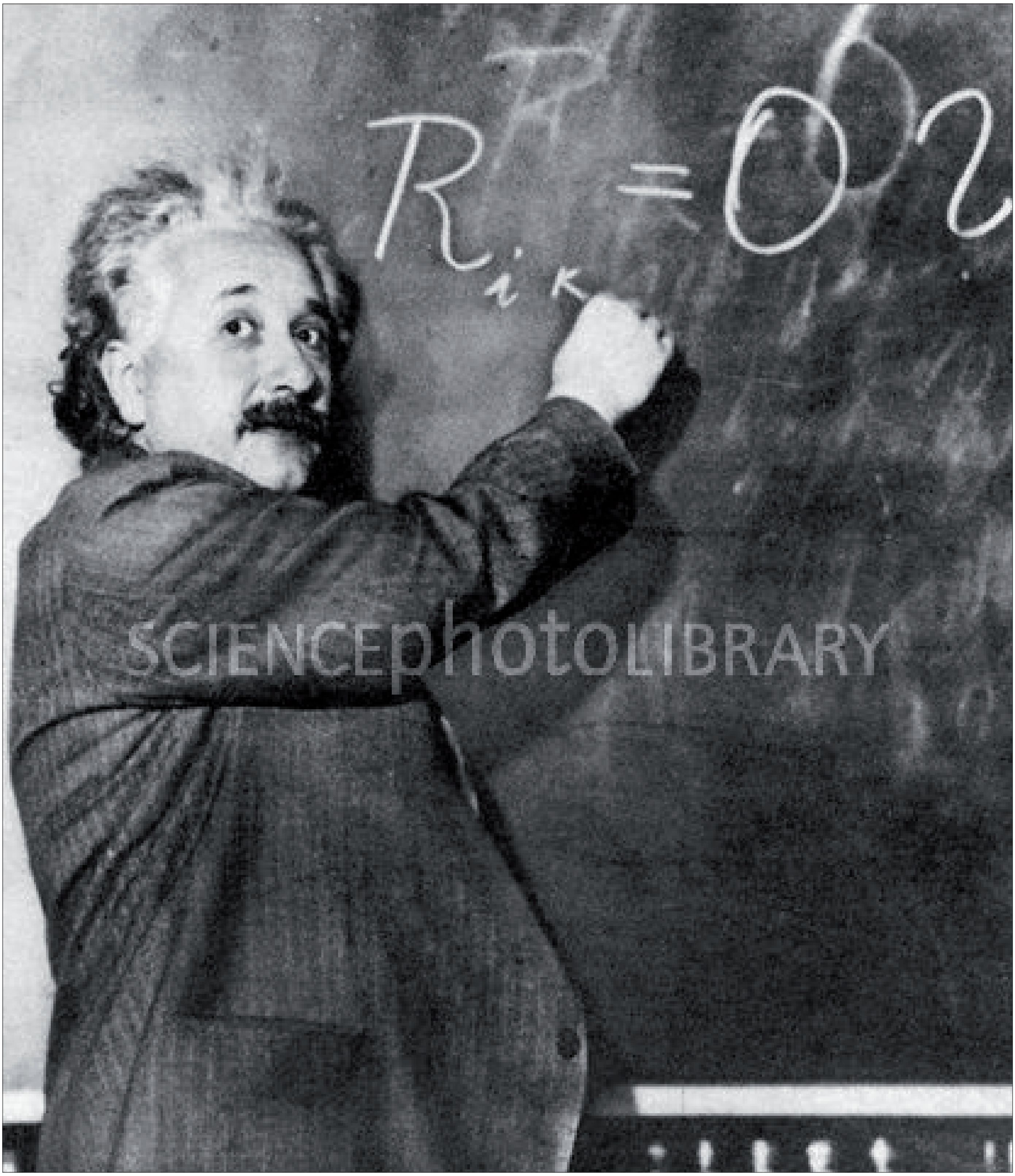
On Einstein's universe

To appreciate my consternation at Einstein's statement above, a little knowledge of his cosmology is required.

Following the successful formulation of his general theory of relativity in 1915, Einstein set about applying his new theory of gravity to the universe as a whole. To his great surprise, he found that relativity predicted a cosmos that is *dynamic*, that is, one that would expand or contract over time. As no evidence for such a phenomenon was known to him, Einstein added a term to the field equations of relativity that could stabilize the universe by counteracting the attractive influence of gravity. This term, the famous "cosmological constant", allowed Einstein to predict a cosmos that was both static and finite, whose radius and average density of matter could be calculated from first principles.

Over the next few years, other theoreticians also explored relativistic models of the cosmos. In particular, the Russian physicist Alexander Friedmann suggested that non-static models of the universe should be considered, and he derived a whole class of dynamic cosmic models from Einstein's field equations in 1922 and 1924. A few years later, the Belgian cleric and theorist Georges Lemaître proposed that emerging observations of a systematic recession of the distant galaxies might constitute evidence of an expansion of space on the largest scales, and, unaware of Friedmann's work, showed that such a cosmic expansion could be derived from Einstein's field equations. Einstein was made aware of the work of both Friedmann and Lemaître, but he found their cosmic models very far-fetched.

All this changed in 1929 with the discovery by Edwin Hubble of a linear relation between the reces-



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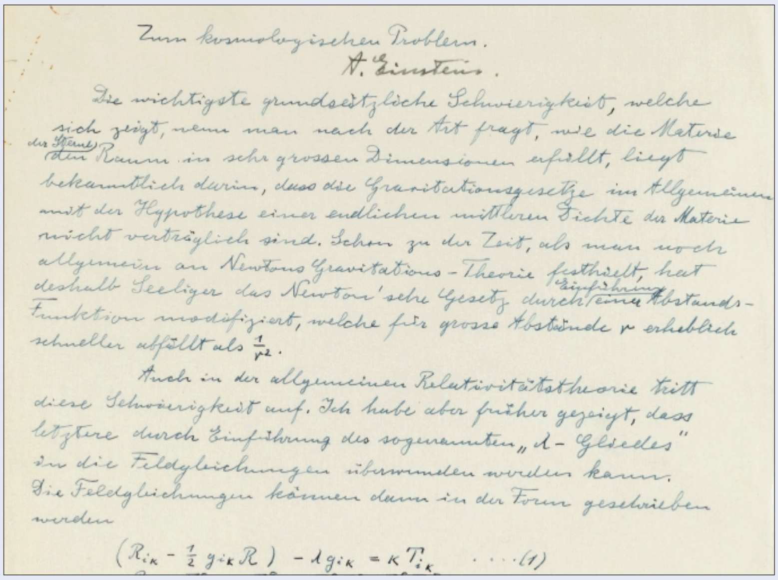
Chalking it up
Albert Einstein at the California Institute of Technology in early 1931. His interest in cosmology was reawakened during this visit.

sion velocity of distant galaxies and their distance from the Sun (*Proc. Natl Acad. Sci.* **15** 168). Many theorists saw the phenomenon as possible evidence for an expansion of space, and set about constructing relativistic models of an expanding universe similar to those of Friedmann and Lemaître. In all these theories, it was assumed that the average density of matter in the universe would decrease as space expanded – what is known as an "evolving" universe.

Einstein himself published two evolving models, one in 1931 (the *Sitzsungsberichte* paper I had been studying) and another with Willem de Sitter in 1932. In both cases, Einstein removed the cosmological constant from the field equations of relativity, commenting that the observed expansion of space had rendered it redundant. In his view, the term he had introduced to counteract gravity – to keep the universe static – was no longer required.

1 An intriguing introduction

Albert Einstein Archives, Hebrew University of Jerusalem



Zum kosmologischen Problem.

A. Einstein.

Die wichtigste grundsätzliche Schwierigkeit, welche sich zeigt, wenn man nach der Art fragt, wie die Materie der Sterne den Raum in sehr grossen Dimensionen erfüllt, liegt bekanntlich darin, dass die Gravitationsgesetze im Allgemeinen mit der Hypothese einer endlichen mittleren Dichte der Materie nicht verträglich sind. Schon zu der Zeit, als man noch allgemein an Newtons Gravitations-Theorie festhielt, hat deshalb Seeliger das Newton'sche Gesetz durch Einführung einer Abstands-Funktion modifiziert, welche für grosse Abstände r erheblich schneller abfällt als $\frac{1}{r^2}$.

Auch in der allgemeinen Relativitätstheorie tritt diese Schwierigkeit auf. Ich habe aber früher gezeigt, dass letztere durch Einführung des sogenannten „λ-Gliedes" in die Feldgleichungen überwunden werden kann. Die Feldgleichungen können dann in der Form geschrieben werden

$$\left(R_{ik} - \frac{1}{2} g_{ik} R\right) - \lambda g_{ik} = \kappa T_{ik} \quad \ldots (1)$$

Shown here is a scan of the title and opening paragraphs of a previously undiscovered manuscript by Albert Einstein, in which he explored a "steady-state" model of the universe – one that expands but in which the average density of matter remains constant.

Einstein's steady-state model and cosmology today

X-ray: NASA/CXC/SAO. Optical: NASA/ESA/Hubble

Evidence found Observations of type Ia supernovae, such as the one that left behind this remnant, reveal the accelerating expansion of the universe.

The recent observation of an accelerated expansion of the universe has focused interest once more on Einstein's cosmological constant, λ. It could be said that Einstein's association of the cosmological constant with an energy of space in the manuscript discussed in the main article anticipates the modern postulate of dark energy ("by setting the λ-term, space itself is not empty of energy").

Another point of intersection with modern cosmology concerns the theory of cosmic inflation; the De Sitter geometry used in today's models of inflation is exactly that used by Einstein in his steady-state model.

Einstein's steady-state theory

Being familiar with Einstein's evolving cosmic models of 1931 and 1932, it soon became clear to my colleague and I that hot afternoon that the manuscript before us constituted something quite different. To confirm this, we set about translating and analysing the document over the next few weeks.

In the introductory section of the manuscript, Einstein proposes an alternative to the evolving models that were proposed in the wake of Hubble's observations. "In what follows," he writes, "I wish to draw attention to a solution to equation (1) that can account for Hubbel's [sic] facts, and in which the density is constant over time." Later on, he suggests a mechanism whereby the density of matter in an expanding universe could remain constant: the continuous formation of matter from empty space. "For the density to remain constant, new particles of matter must be continually formed within that volume from space."

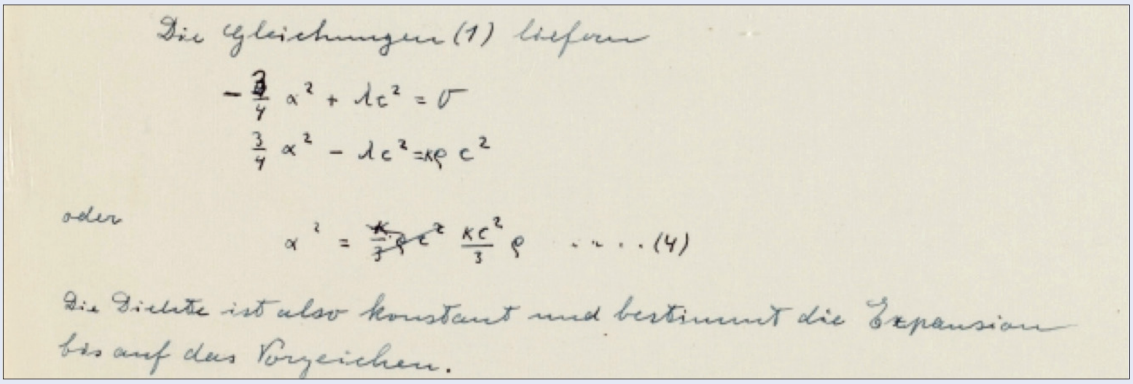
Thus, Einstein is exploring the possibility of a universe that expands but remains in a steady state due to a continuous replenishment of matter. (The term "steady state" is used in many areas of physics to describe a system that is dynamic but essentially unchanging.) Most intriguingly, Einstein links the continuous creation of matter with the cosmological constant: "The conservation law is preserved in that, by setting the λ-term, space itself is not empty of energy; its validity is well known to be guaranteed by equations (1)."

Until now, it had been entirely unknown that Einstein once explored the idea of a steady-state model of the universe. Why has this fact remained hidden? More to the point, why was this manuscript never published? A clue may be found by considering the equations in figure 2. Starting with the De Sitter metric of space–time geometry, Einstein derives from the field equations a relation (equation 4) between the average matter density of the universe, ρ, and the expansion coefficient, α, a constant. This is a stunning result, as it implies that the density of matter is directly related to the expansion coefficient and remains constant. However, there is a problem: a cursory inspection of the simultaneous equations in figure 2 suggests that they in fact lead to the trivial solution $\rho = 0$, in other words, to a universe empty of matter, rather than equation (4). It appears that this null result was initially masked by an error in Einstein's earlier derivation of the simultaneous equations; close scrutiny of figure 2 shows that the coefficient of α^2 in the first equation was originally 9/4, giving equation (4), but this coefficient was later amended to –3/4.

So what was going on? At this point, we decided to reconstruct Einstein's steady-state theory from first principles, with the help of Werner Nahm of the Dublin Institute for Advanced Studies. Sure enough, our analysis showed that the coefficient of α^2 in the first equation should indeed be –3/4, leading to a null solution for the density of matter. From figure 2, it seems reasonable to conclude that Einstein found his error on revision, realized that the model led to a trivial solution, and set it aside.

With the benefit of today's mathematics, a modern cosmologist might point out that Einstein's steady-state model could only lead to a null solution. This is because he neglected to introduce a specific term to the field equations representing the continuous creation of matter, instead loosely associating the process with the cosmological constant. We find it very interesting that Einstein did not see this problem from the first – and when he did, he apparently abandoned the model rather than modify the field equations.

2 A universe of constant density



Die Gleichungen (1) liefern

$$-\tfrac{3}{4}\alpha^2 + \lambda c^2 = 0$$

$$\tfrac{3}{4}\alpha^2 - \lambda c^2 = \kappa\rho c^2$$

oder

$$\alpha^2 = \frac{\kappa c^2}{3}\rho \quad \ldots (4)$$

Die Dichte ist also konstant und bestimmt die Expansion bis auf das Vorzeichen.

$$-\tfrac{3}{4}\alpha^2 + \lambda c^2 = 0$$

$$\tfrac{3}{4}\alpha^2 - \lambda c^2 = \kappa\rho c^2$$

$$\alpha^2 = \frac{\kappa c^2}{3}\rho \qquad (4)$$

In equation 4 from Albert Einstein's previously undiscovered manuscript, the quantities α, κ and c are constants. The equation therefore implies a direct relation between ρ, the average matter density of the universe, and α, the expansion coefficient. Einstein explains the implication of this below the equation: "The density is therefore constant and determines the expansion apart from its sign."

Dating the manuscript

How does Einstein's manuscript fit in with his other models of the cosmos? When was it written? While it is assigned the year 1931 in the Albert Einstein Archives, this date is worth reviewing as the document was until now mistaken for a draft of his 1931 *Sitzungsberichte* paper.

From the reference to Hubble's observations, one can certainly surmise that the manuscript was written after 1929. On the other hand, it is unlikely that the document was penned after 1931, as there is no reference to Einstein's published cosmic models of April 1931 or 1932, or to Lemaître's 1931 postulate of an explosive origin for the cosmos (*Nature* **127** 706). It is known from Einstein's diaries that his interest in cosmology was reawakened during a three-month stay at the California Institute of Technology (Caltech), US, in early 1931, in particular by his interactions with the Caltech relativist Richard Tolman and with the astronomers at the neighbouring Mount Wilson Observatory. Thus it seems very likely that the manuscript was penned during this period. (My colleagues and I also note that the document is written on American notepaper.) If this dating is correct, the manuscript represents Einstein's very first attempt at a model of the universe in the wake of emerging evidence for a cosmic expansion.

Later steady-state theories

As is well known, the notion of a steady-state universe was also explored by a trio of physicists at the University of Cambridge in the late 1940s. Concerned by several problems associated with evolving models, Fred Hoyle, Hermann Bondi and Thomas Gold all considered the idea of an expanding universe that remains unchanged due to a continuous creation of matter. While it is extremely unlikely that the trio were aware of Einstein's earlier attempt, it is intriguing that Hoyle's formulation of a relativistic steady-state model resembles that of Einstein's manuscript in many respects – with the crucial difference that Hoyle added a "creation term" to the field equations in order to represent the continuous creation of matter. Hoyle's steady-state theory posed a controversial alternative to evolving models of the cosmos for some years, although it was eventually ruled out by experiment (notably, by the study of the distribution of galaxies at different epochs and the detection of a universal background radiation emanating from the early universe).

Thus, it could be argued that steady-state models of the cosmos are of little interest today – even an attempt by Albert Einstein. However, we find Einstein's manuscript very interesting from the point of the view of the evolution of ideas. Indeed, it is a fundamental tenet of historical research that unsuccessful ideas can be of great importance in understanding how theories develop. In this case, it now appears that, when first confronted with evidence for a cosmic expansion, Einstein conducted an internal debate between steady-state and evolving models of the cosmos, decades before a similar debate engulfed the cosmological community. This fits well with Einstein's philosophical attachment to an unchanging, static universe in 1917 and his hostility to the evolving models of Friedman and Lemaître when they were first proposed in the 1920s. On the other hand, it is also interesting that when Einstein realized that his steady-state model didn't work, he turned to evolving cosmic models of the cosmos rather than attempt a more contrived steady-state theory by amending the field equations.

All in all, Einstein's manuscript reminds us that today's model of the evolving universe did not occur as a sudden "paradigm shift", but as a slow process of discovery in both theory and observation. No doubt, more light will be shed on this era as the process of collating and digitizing Einstein's original papers continues – an invaluable project run by the Einstein Papers Project at Caltech in conjunction with the Hebrew University of Jerusalem. ■

- Einstein's manuscript is available online at http://alberteinstein.info/vufind1/Record/EAR000034354. A full translation and analysis is given in the paper "Einstein's steady-state theory: an abandoned model of the cosmos" by Cormac O'Raifeartaigh, Brendan McCann, Werner Nahm and Simon Mitton (2014 *Eur. Phys. J. H* **39** 1)